Part 3

Mutual Inductance

Main Outlines

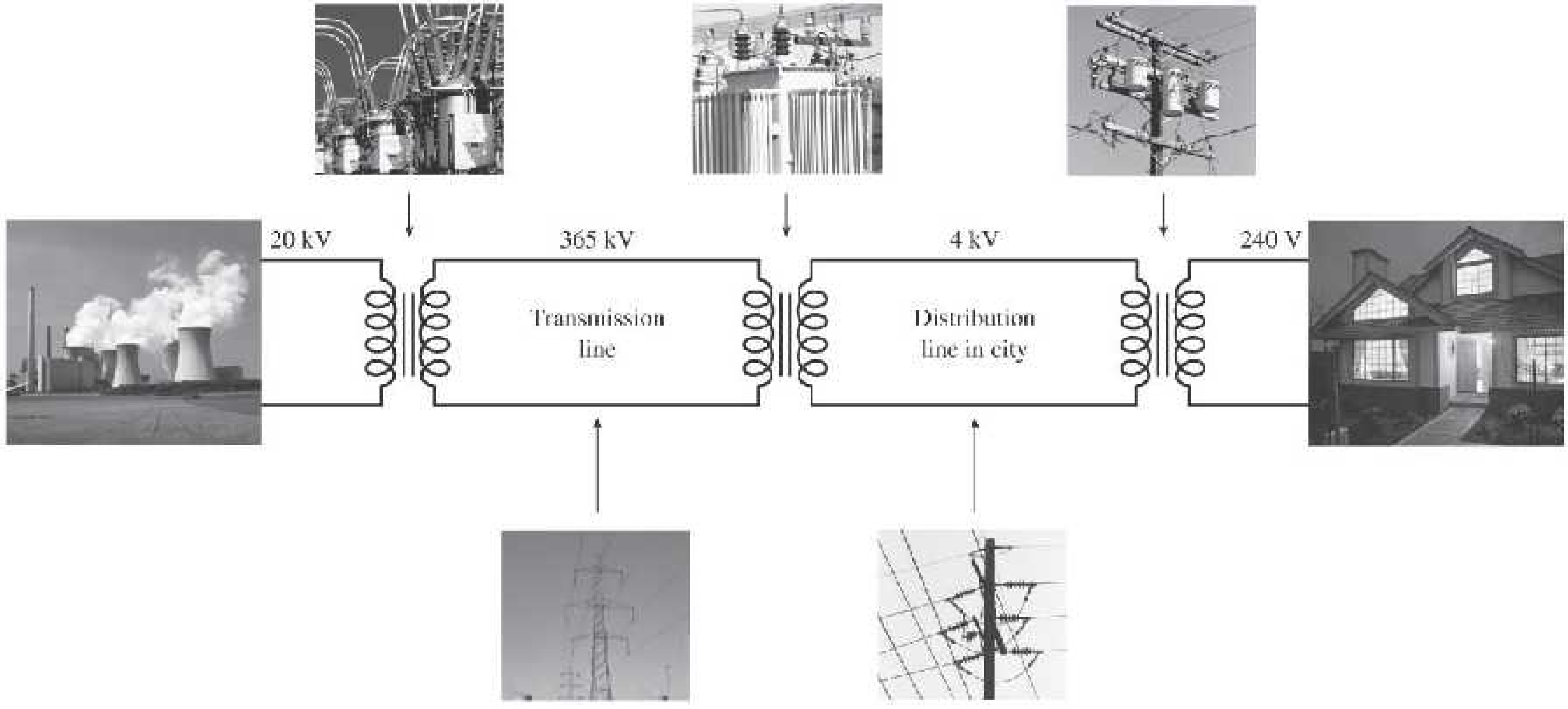
- Review of self inductance
- Concept of mutual inductance
- Mutual inductance in terms of self inductance
- Polarity of the mutually induced voltages (Dot Convention)
- Procedure for determining dot marking
- Use of dot markings in circuit analysis
- Energy calculations

Magnetically Coupled Circuits

- When two loops with or without contacts between them **affect each other** through the magnetic field generated by one of them, it called ***magnetically coupled***
- **Example: transformer**
  - ✓An electrical device designed on the basis of the concept of magnetic coupling
  - ✓ Used magnetically coupled coils to transfer energy from one circuit to another

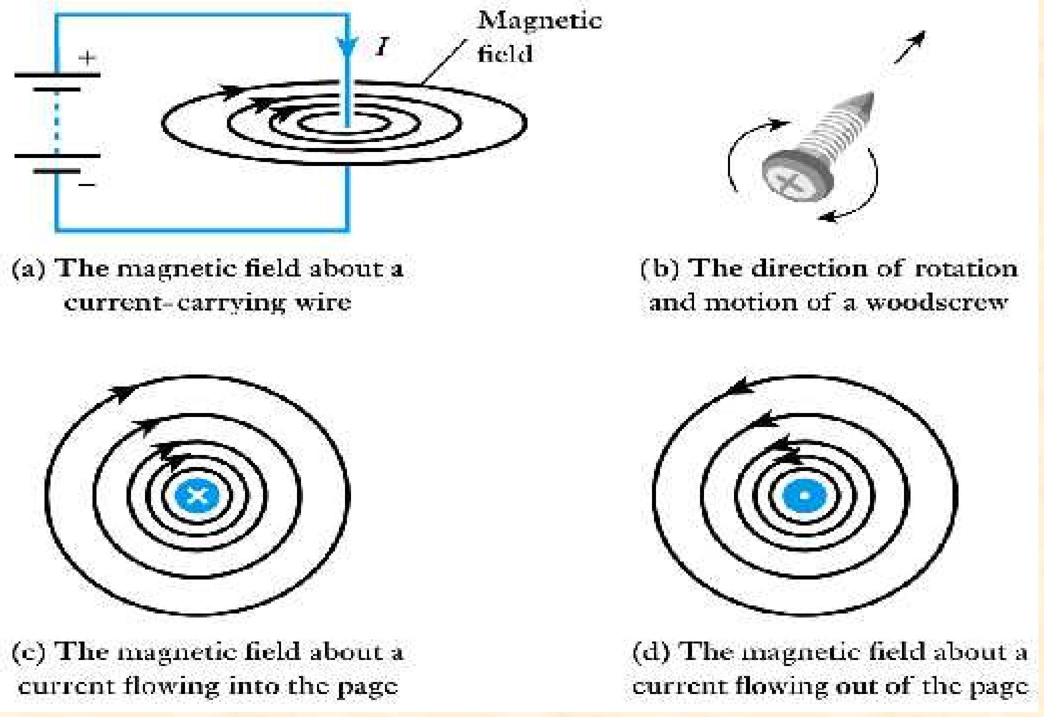
Transformers and Power Transmission

- Electric power is most efficiently transmitted at high voltages.
  - This reduces I^2R energy losses in the power lines.
  - But most end uses require lower voltages.
  - Transformers accomplish voltage changes throughout the power grid.



Magnetic Field

- A wire carrying a current *I* causes a **magneto-motive force (m.m.f) *F***
  - this produces a **magnetic field**
  - ***F*** has units of Amperes
  - for a single wire *F* is equal to *I*



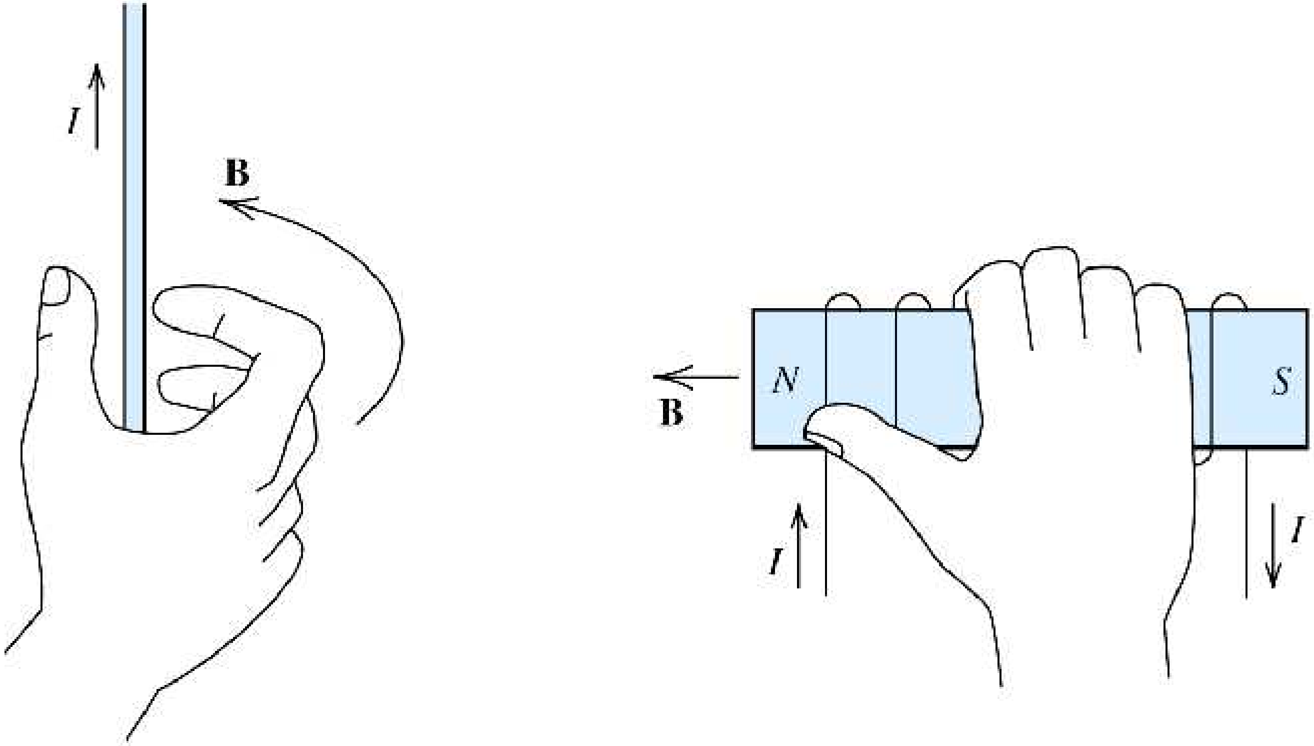
(a) The magnetic field about a current-carrying wire

(b) The direction of rotation and motion of a woodscrew

(c) The magnetic field about a current flowing into the page

(d) The magnetic field about a current flowing out of the page

Magnetic Field



(a) If a wire is grasped with the thumb pointing in the current direction, the fingers encircle the wire in the direction of the magnetic field

(b) If a coil is grasped with the fingers pointing in the current direction, the thumb points in the direction of the magnetic field inside the coil

Right-Hand Rule

Magnetic Reluctance

- In a *resistive* circuit, the resistance is a measure of how the circuit opposes the flow of electricity
- In a *magnetic* circuit, the **reluctance**, $\Re$ is a measure of how the circuit opposes the flow of magnetic flux

✓ In a resistive circuit $R = V/I$

✓ In a magnetic circuit $\Re = \dfrac{F}{\Phi}$

- The units of reluctance are amperes per weber (A/ Wb)
- The magnetic **Permeance** is given by: $P = \dfrac{1}{\Re}$

Flux Linkages and Faraday's Law

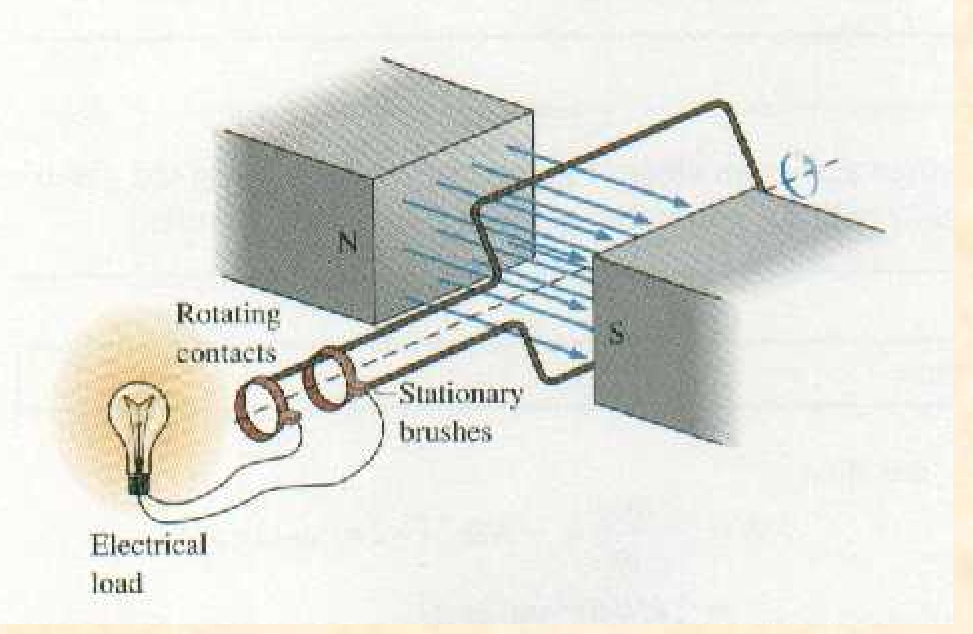
- **The flux linking a coil with N turns:** $\lambda = N\Phi$

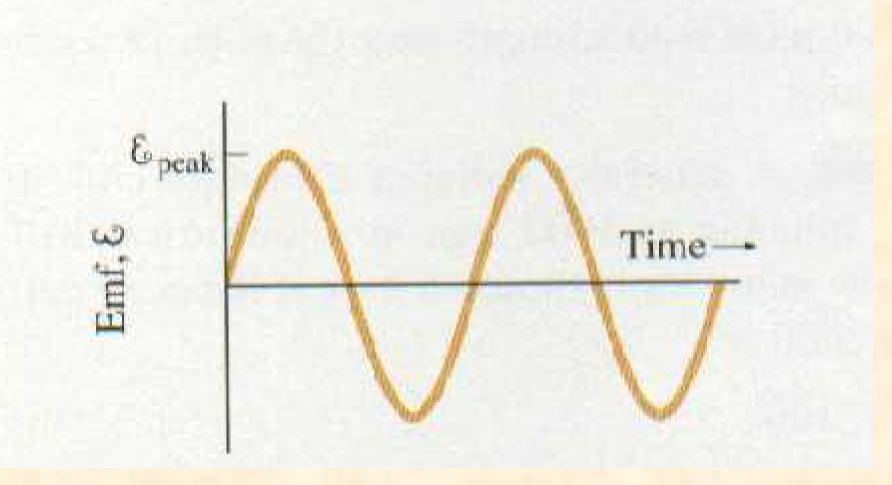
- **Faraday's law of magnetic induction:** $e = \dfrac{d\lambda}{dt}$

- The voltage induced in a coil whenever its flux linkages are changing.

- Changes occur from:
  - Magnetic field changing in time
  - Coil moving relative to magnetic field

Faraday's Law



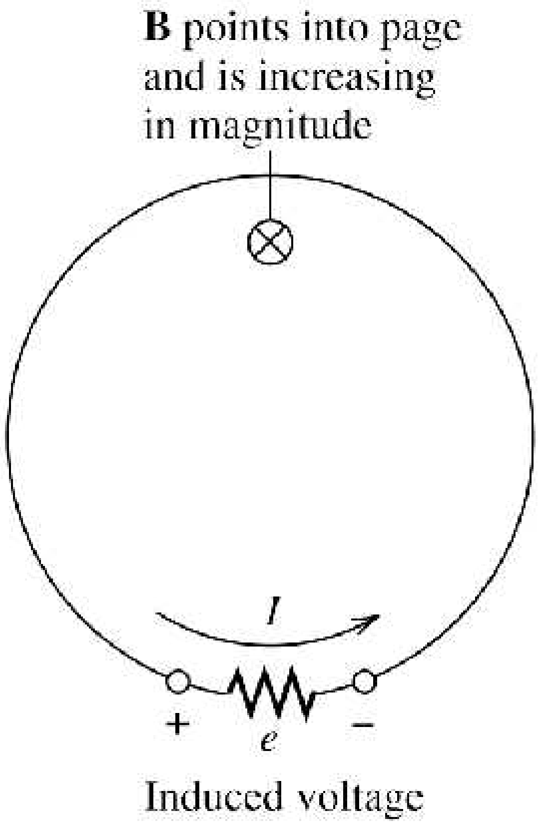


Lenz's Law

- **Lenz's law** states that the polarity of the induced voltage is such that the voltage would produce a current (through an external resistance) that opposes the original change in flux linkages.

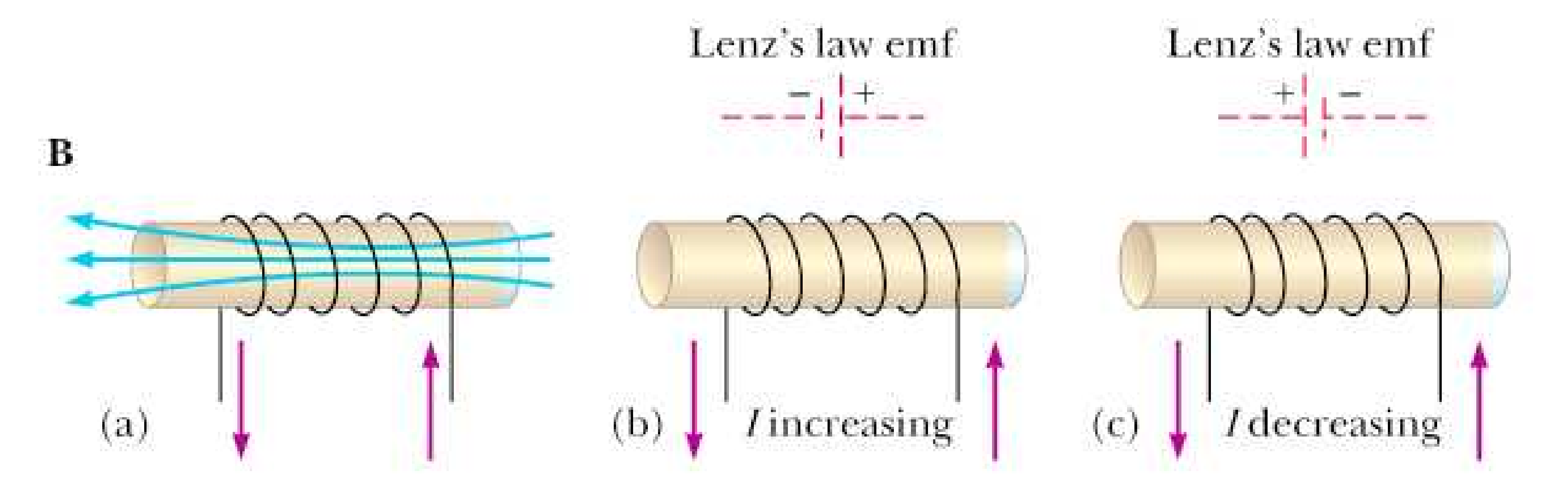
- The current in a conductor, as a result of an induced voltage, is such that the magnetic flux due to it is **opposite to** the magnetic flux that caused the induced voltage

Lenz's Law



When the flux linking a coil changes, a voltage is induced in the coil. The polarity of the voltage is such that if a circuit is formed by placing a resistance across the coil terminals, the resulting current produces a field that tends to oppose the original change in the field.

Lenz's Law



➢ When *I* changes, an emf is induced in the coil

➢ If ***I* is increasing** (and therefore increasing the flux through the coil), then the induced emf will set up a magnetic field to oppose the **increase** in the magnetic flux in the direction shown.

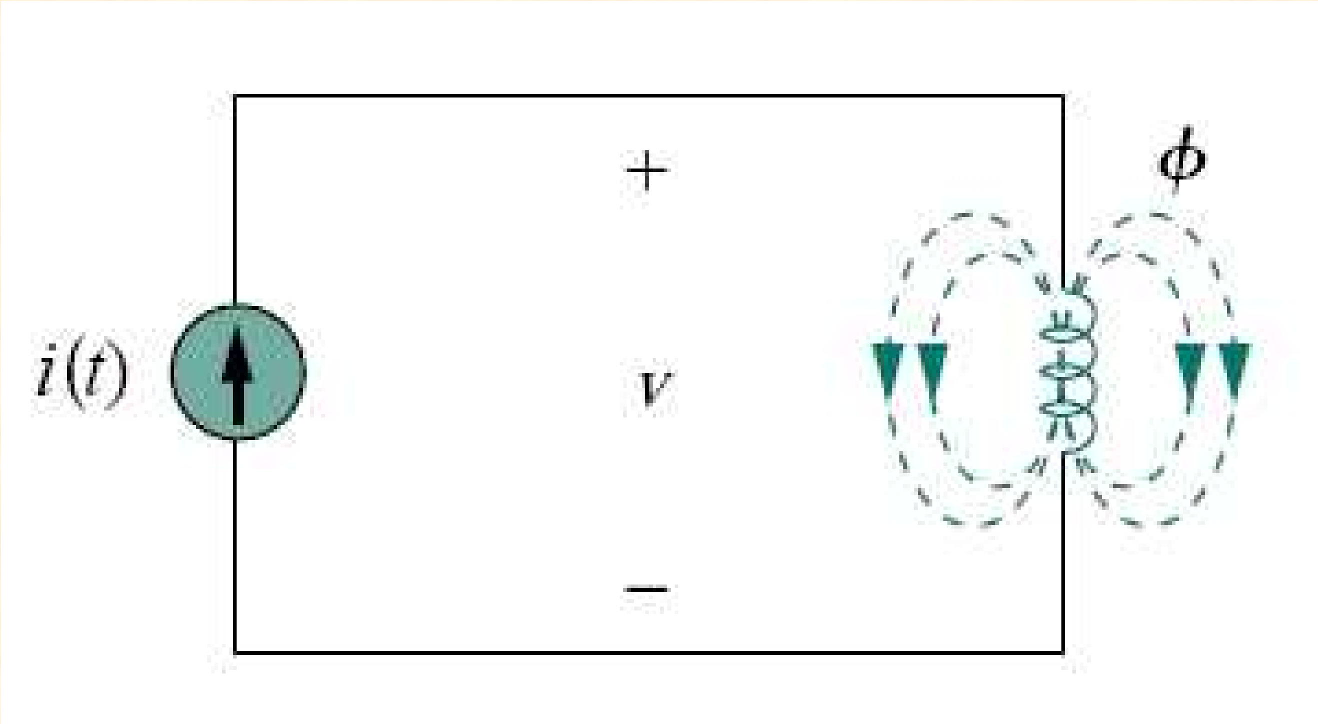
➢ If ***I* is decreasing**, then the induced emf will set up a magnetic field to oppose the **decrease** in the magnetic flux.

Self and Mutual Inductance

- 1 coil (inductor)
  - Single solenoid has only self-inductance (L)

- 2 coils (inductors)
  - 2 solenoids have self-inductance (L) & Mutual-inductance (M)

Self Inductance



✓ A coil with N turns produced ϕ = magnetic flux

✓ Only has self inductance, L

Self Inductance

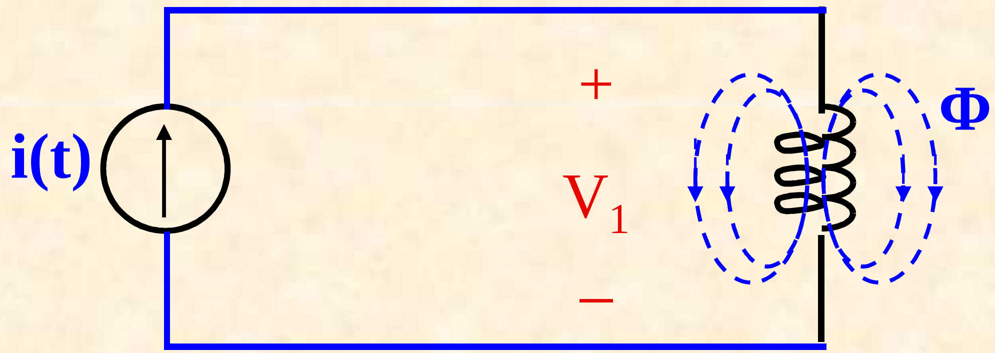
- It called **self inductance** because it relates the voltage induced in a coil by a time varying current in the same coil

- Consider a single inductor with **N** number of turns when current, ***i*** flows through the coil, a magnetic flux, **Φ** is produces around it

$$\text{¤} = \frac{(N\,i)}{\Re} = (N\,i)\,P$$

$$\text{a} = N\,\text{¤} = N\,\frac{(N\,i)}{\Re}$$

$$\text{a} = \left(\frac{N^2}{\Re}\right) i = \left(N^2 P\right) i$$



$$L = \frac{N^2}{\Re} = N^2\,P$$

$$\text{a} = N\,\text{¤} = L\;i$$

Self Inductance

- According to Faraday's Law, the voltage, (***v***) induced in the coil is <u>proportional to</u> (***N***) number of turns and rate of change of the magnetic flux, ***Φ*** ;

$$v = N \frac{d\Phi}{dt}$$

- In addition, the induced voltage, (***v***) can be written in terms of the self inductance, (***L***) and rate of change of the current, (***i***);

$$v = L \frac{di}{dt}$$

Self Inductance **(another form)**

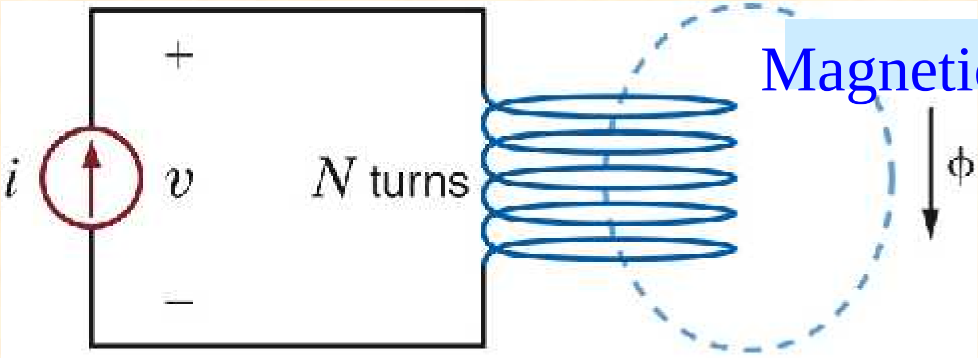
$$v = N \frac{d\text{¤}}{dt}$$

$$\frac{d\text{¤}}{dt} = \frac{d\text{¤}}{di}\frac{di}{dt}$$

$$v = N \frac{d\text{¤}}{di}\frac{di}{dt} \qquad \text{or} \qquad v = L \frac{di}{dt}$$

$$L = N \frac{d\text{¤}}{di} \qquad (\mathrm{H})$$

Self Inductance (conclusions)



$$\lambda = N\phi$$

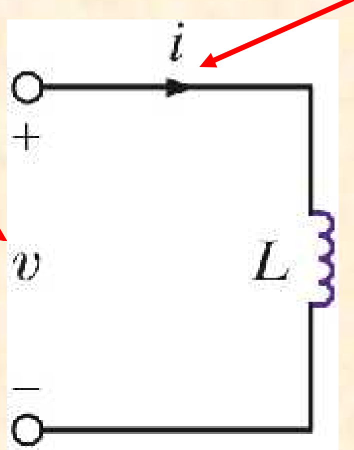
Total magnetic flux linked by N-turn coil

$$v = \frac{d\lambda}{dt}$$

Faraday's Induction Law

$$\lambda = Li$$

Ampere's Law

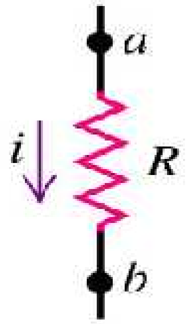


Ideal Inductor

$$v = L\frac{di}{dt}$$

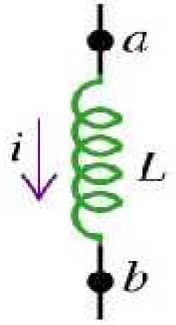
$$L = \frac{N^2}{\Re} = N^2 P$$

Resistor and Inductor



$$V_{ab} = iR$$

(a) Resistor with current i flowing from a to b: potential drops from a to b



$$V_{ab} = L\frac{di}{dt}$$

(b) Inductor with current i flowing from a to b:
- If $di/dt > 0$: potential drops from a to b
- If $di/dt < 0$: potential increases from a to b
- If i is constant ($di/dt = 0$): no potential difference

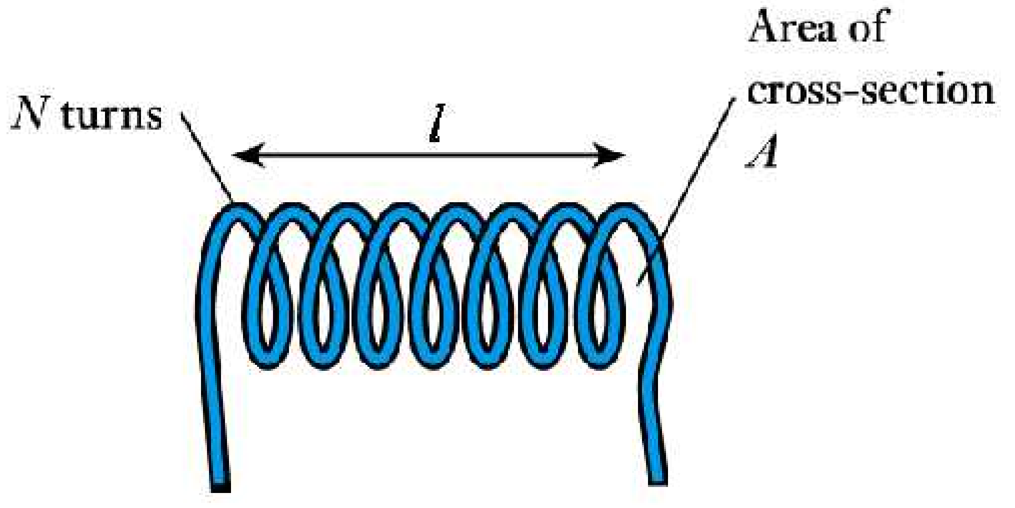
Potential difference across a resistor depends on the current

Potential difference across an inductor depends on the **rate of change** of the current

Inductor

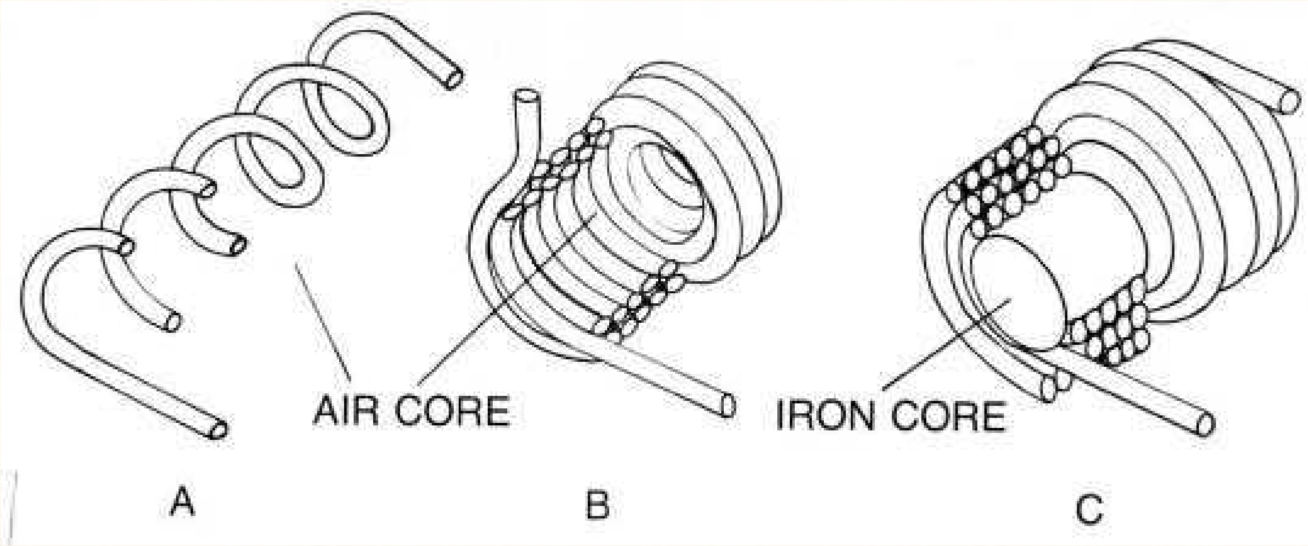
- The inductance of a coil depends on its dimensions and the materials around which it is formed

$$L = \frac{\mu_0 A N^2}{l}$$



(a) An air-filled coil

Types of Inductors



$$L = \frac{\mu_0 \mu_r A N^2}{l} = \frac{N^2}{\Re} = N^2 P$$

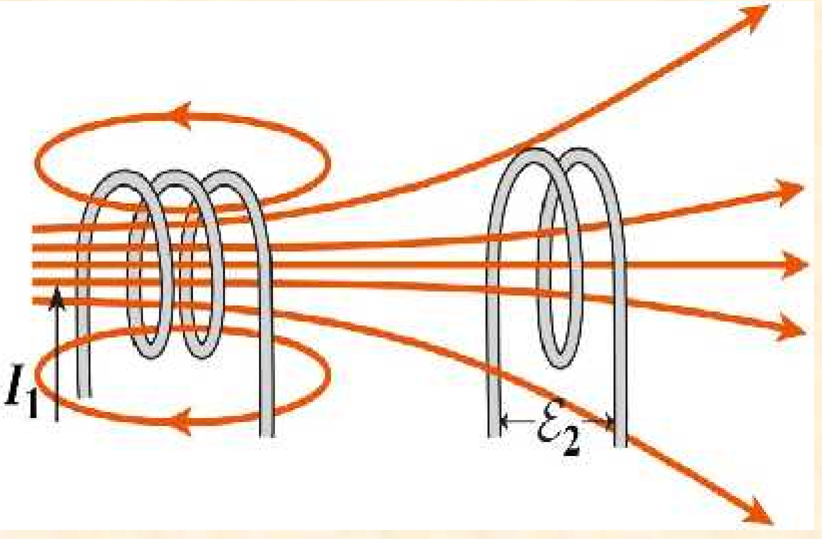
$$\Re = \frac{1}{P} = \frac{l}{\mu_0 \mu_r A}$$

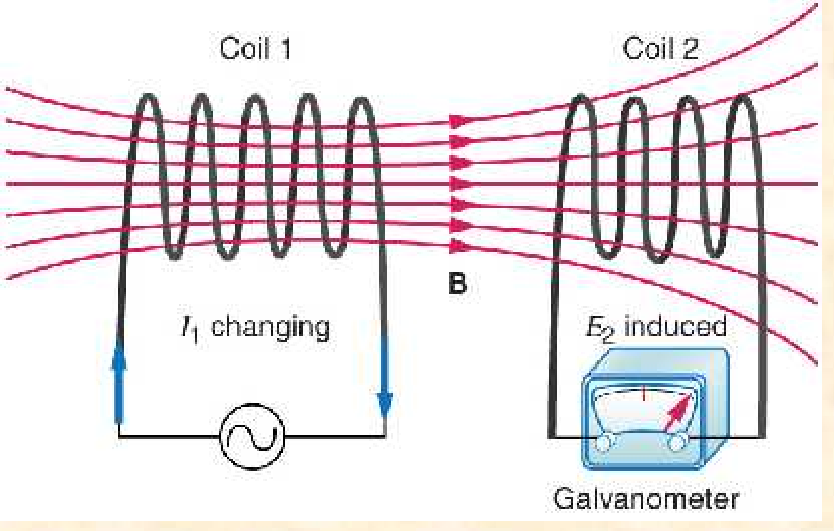
Factors Affecting Inductance of Coils

- ✓ **Numbers of Turns-** Inductance varies directly with the square of the number of turns
- ✓ **Permeability of Core-** Inductance varies directly with the permeability of the core
- ✓ **Cross-sectional Area of Core-** Inductance varies directly with the cross-sectional area of the core
- ✓ **Length of Core-** Inductance varies inversely with the length of the core

Mutual Inductance

- When two inductors or coils are in close proximity to each other, magnetic flux **caused** by current in one coil **links** with the other coil, therefore producing the induced voltage



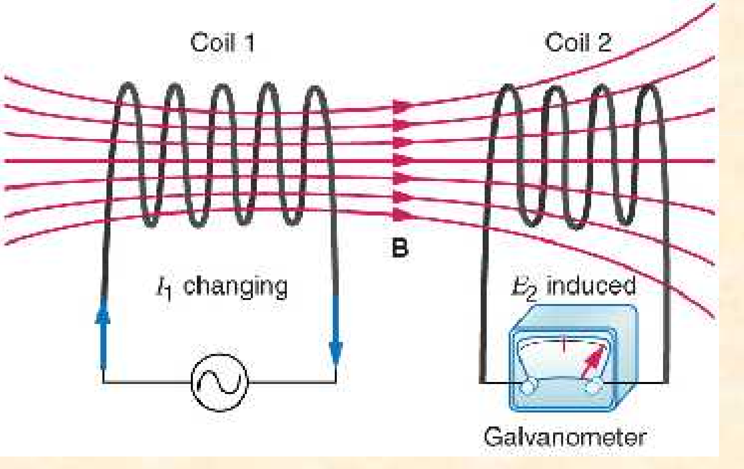


Mutual Inductance

- **Mutual inductance** occurs when a changing current in one circuit results, via changing magnetic flux, in an induced emf and thus a current in an adjacent circuit

  - The coils are said to have mutual inductance M, which can either add or subtract from the total inductance depending on if the fields are aiding or opposing

  - **Mutual inductance** is the ability of one inductor to induce a voltage across a neighboring inductor

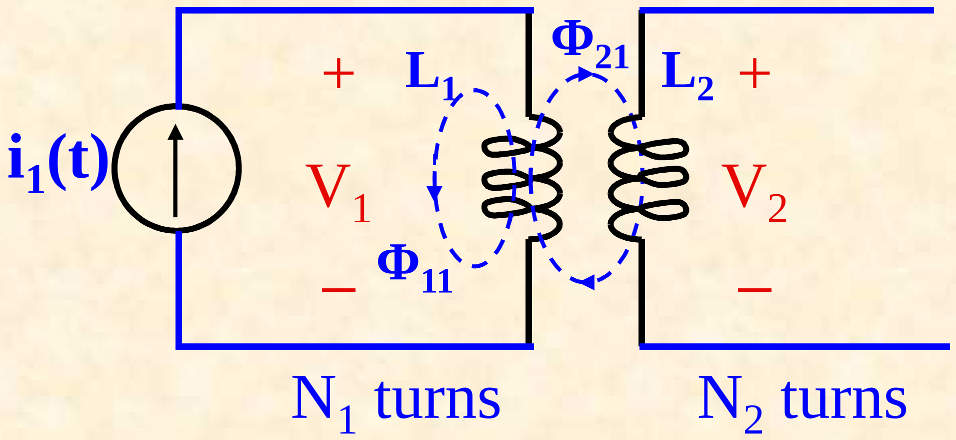


Mutual Inductance

Consider the following two cases:

- **Case 1:**

two coil with self – inductances L_1 and L_2 which are in close proximity which each other. Coil 1 has N_1 turns, while coil 2 has N_2 turns



Mutual Inductance

- Magnetic flux $\mathbf{\Phi_1}$ from coil 1 has two components;
  - * Φ_{11} links only coil 1
  - * Φ_{21} links both coils

✓ Hence; $\Phi_1 = \Phi_{11} + \Phi_{21}$

Leakage flux

$$\text{¤}_{11} = \frac{N_1 i_1}{\Re_{11}} = N_1 i_1\ P_{11}$$

where

$$\text{¤}_1 = \frac{N_1 i_1}{\Re_1} = N_1 i_1\ P_1$$

Total flux

$$\text{¤}_{21} = \frac{N_1 i_1}{\Re_{21}} = N_1 i_1\ P_{21}$$

Magnetizing (Mutual) flux

$$P_1 = P_{11} + P_{21}$$

Mutual Inductance

➢ Thus; the voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_1}{dt}$$

$$v_1 = N_1 \frac{d}{dt}(\phi_{11} + \phi_{21})$$

$$v_1 = N_1^2 (P_{11} + P_{21}) \frac{di_1}{dt}$$

$$v_1 = \left(N_1^2 P_1\right) \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

Mutual Inductance

✓ The Voltage induces in coil 2

$$v_2 = N_2 \frac{d¤_{21}}{dt}$$

$$¤_{21} = \frac{N_1 i_1}{\Re_{21}} = N_1 i_1 \, P_{21}$$

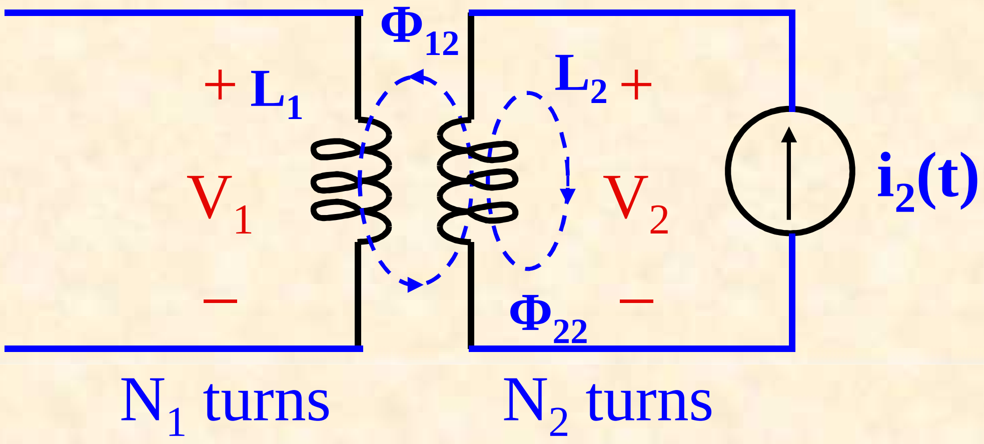
$$v_2 = N_2 N_1 \, P_{21} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

Subscript 21 in M_{21} means the mutual inductance on coil 2 due to coil 1

$$M_{21} = \frac{N_2 N_1}{\Re_{21}} = N_2 N_1 \, P_{21}$$

Mutual Inductance

- **Case 2:** Same circuit but let current i_2 flow in coil 2.



✓ The magnetic flux Φ_2 from coil 2 has two components:

*** Φ_{22} links only coil 2**

*** Φ_{12} links both coils**

Mutual Inductance

➤ Hence; $\Phi_2 = \Phi_{22} + \Phi_{12}$

where

Leakage flux

$$\text{¤}_{22} = \frac{N_2 i_2}{\Re_{22}} = N_2 i_2 \; P_{22}$$

$$\text{¤}_{2} = \frac{N_2 i_2}{\Re_{2}} = N_2 i_2 \; P_{2}$$

Total flux

$$\text{¤}_{12} = \frac{N_2 i_2}{\Re_{12}} = N_2 i_2 \; P_{12}$$

Magnetizing (Mutual) flux

$$P_2 = P_{22} + P_{12}$$

Mutual Inductance

✓ Thus; the voltage induced in coil 2

$$v_2 = N_2 \frac{d\phi_2}{dt}$$

$$v_2 = N_2 \frac{d}{dt}(\phi_{22} + \phi_{12})$$

$$v_2 = N_2^2 (P_{22} + P_{12}) \frac{di_2}{dt}$$

$$v_2 = \left(N_2^2 P_2\right) \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

Mutual Inductance

✓ The Voltage induces in coil 1

$$v_1 = N_1 \frac{d\phi_{12}}{dt}$$

$$\phi_{12} = \frac{N_2 i_2}{\Re_{12}} = N_2 i_2 \ P_{12}$$

$$v_1 = N_1 N_2 \ P_{21} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

Subscript 12 in M_{12} means the mutual inductance on coil 1 due to coil 2

$$M_{12} = \frac{N_1 N_2}{\Re_{12}} = N_1 N_2 \ P_{12}$$

Mutual Inductance

- **For a linear system**

$$\Re_{21} = \Re_{12}$$

$$\boxed{M_{21} = M_{12} = M}$$

- Mutual inductance **M** is measured in Henrys (H)

Mutual Inductance (another form)

- **Case 1:**

$$v_1 = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt}$$

$$v_2 = N_2 \frac{d\phi_{21}}{di_1} \frac{di_1}{dt} = M_{21} \frac{di_1}{dt}$$

- **Case 2:**

$$v_2 = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{12}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$